

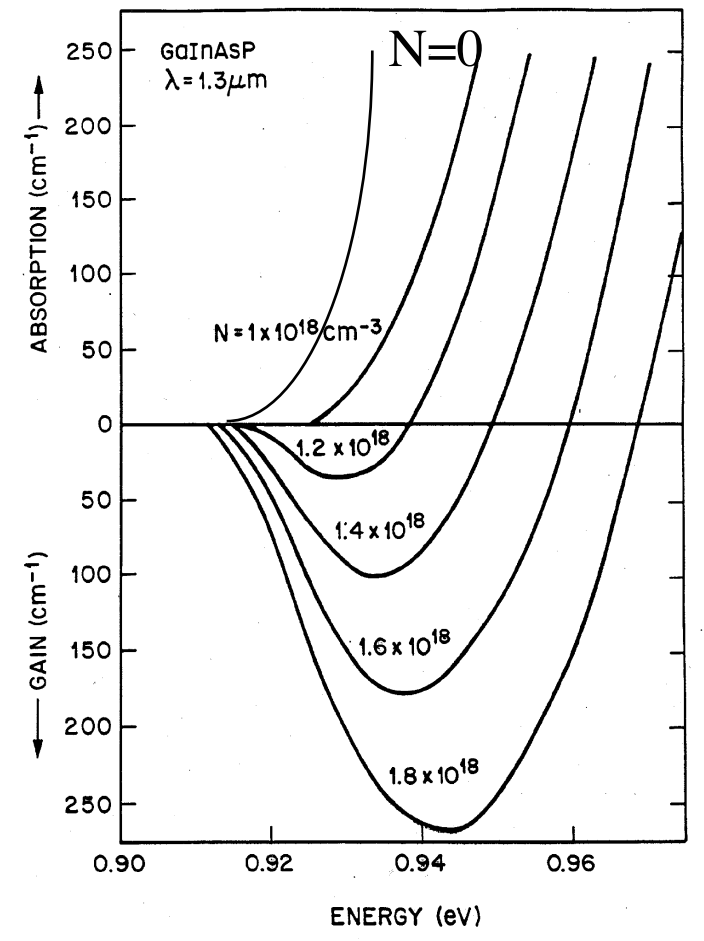


Lecture 3

Material Gain

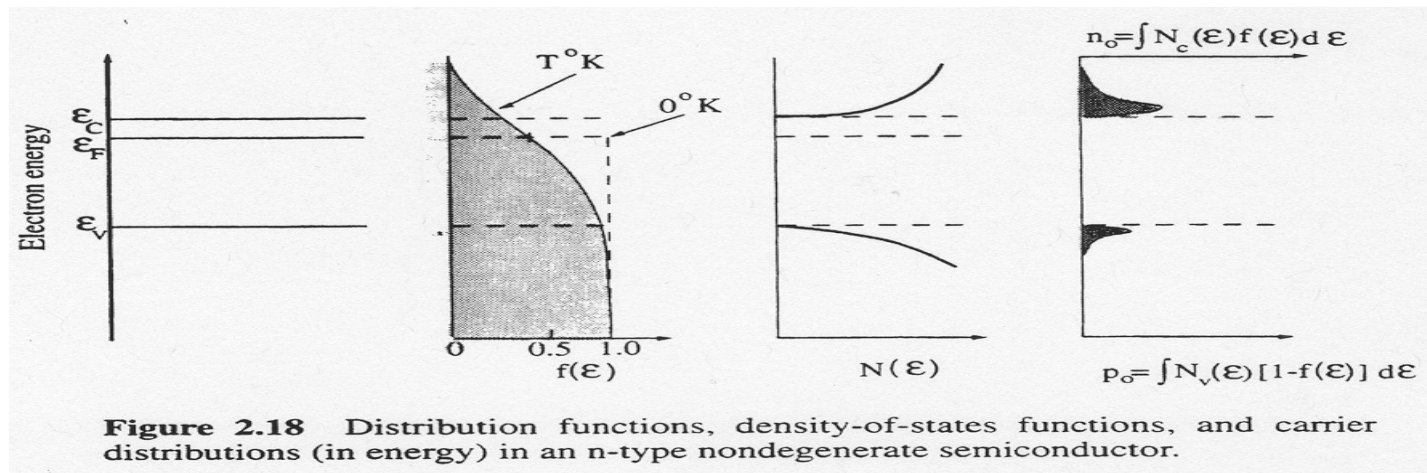
Calculated gain curves
for InGaAsP/InP laser
operating at $1.3\mu\text{m}$

- Gain peak moves to shorter wavelengths with higher pumping
- Higher differential gain for wavelengths shorter than the gain peak



N.K.. Dutta, J. Appl. Phys., **51**, 6095 (1980)

Carrier Density



Density of electrons with
energies between E and $E + dE$

$$N_c = \frac{I\tau}{eV} = \int_0^\infty \rho_c(E) f_c(E) dE$$

$$= \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} \int_0^\infty \frac{E^{1/2}}{e^{(E-E_{Fc})/k_B T} + 1} dE$$

Quasi-Fermi Level

$$E_{Fc}(T = 0) = \left(3\pi^2 \right)^{2/3} \frac{\hbar^2}{2m_c} N_c^{2/3}$$

Optical Gain in Semiconductors (1)

- For plane wave propagation in a complex medium, with k_0 the free space wave vector and n' and n'' the real and imaginary part of the refractive index respectively

$$\beta = k_0 (n' + jn'')$$

$$k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

- The optical gain (loss) for a plane wave propagating in a semiconductor in the z-direction can be approximated by

$$g = -\alpha = \frac{1}{I} \frac{dI}{dz} = 2k_0 n''$$

Optical Gain in Semiconductors

- The band structure and electron probability distribution is given by

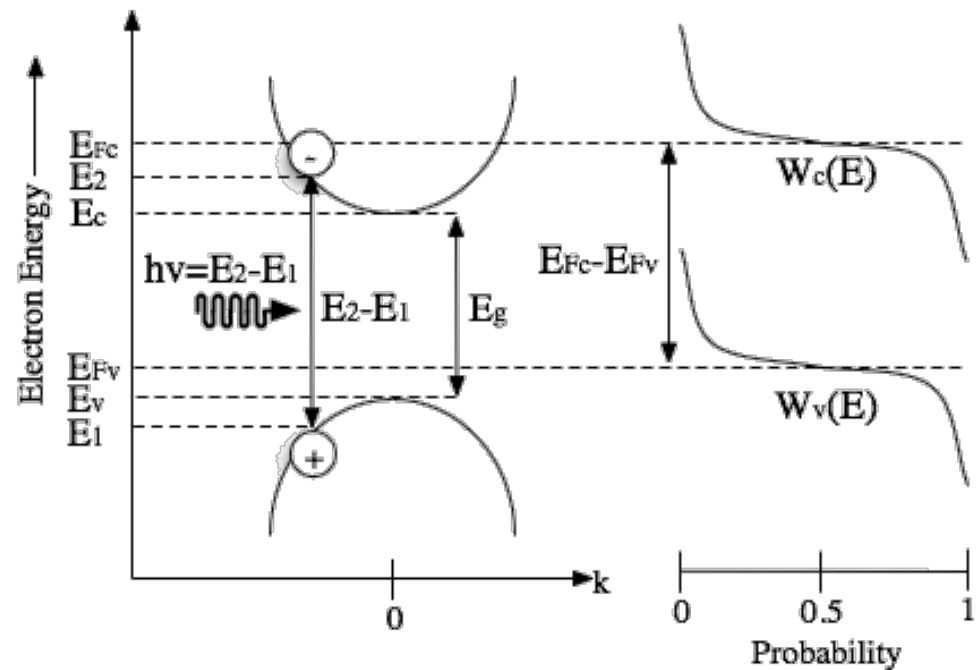
$$W_c(E) = \left[1 + \exp\left(\frac{E - E_{Fc}}{k_B T}\right) \right]^{-1}$$

$$W_v(E) = \left[1 + \exp\left(\frac{E - E_{Fv}}{k_B T}\right) \right]^{-1}$$

$$N = N_c \frac{2}{\pi} \int_{E_c}^{\infty} Z_c(E) W_c(E) dE$$

$$P = N_v \frac{2}{\pi} \int_{-\infty}^{E_v} Z_v(E) [1 - W_v(E)] dE$$

↑
↑
 Carrier density Density of states



Photon Energy and Bandgap

Energy of Photon

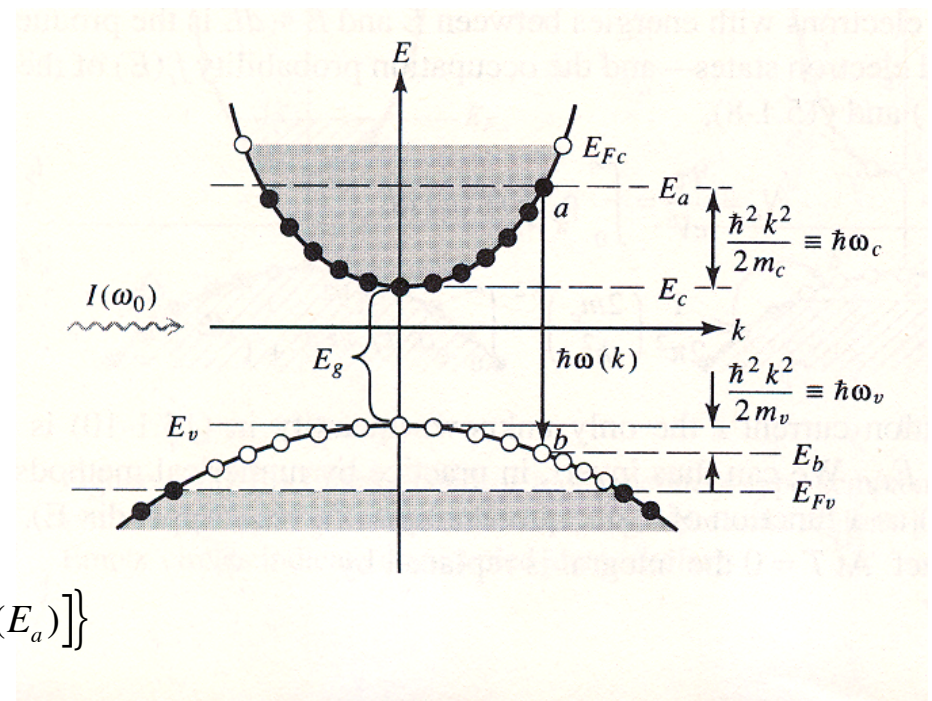
$$E_a - E_b = \hbar\omega = E_g + \frac{\hbar^2 k^2}{2m_c} + \frac{\hbar^2 k^2}{2m_v}$$

For an electron in upper state a and potential lower state b, the downward rate of transition is

$$R_{a \rightarrow b} \propto f_c(E_a) [1 - f_v(E_b)]$$

Effective inversion due to electrons and holes within dk

$$\begin{aligned} N_2 - N_1 &\rightarrow \frac{\rho(k)dk}{V} \left\{ f_c(E_a) [1 - f_v(E_b)] - f_v(E_b) [1 - f_c(E_a)] \right\} \\ &= \frac{\rho(k)dk}{V} [f_c(E_a) - f_v(E_b)] \end{aligned}$$

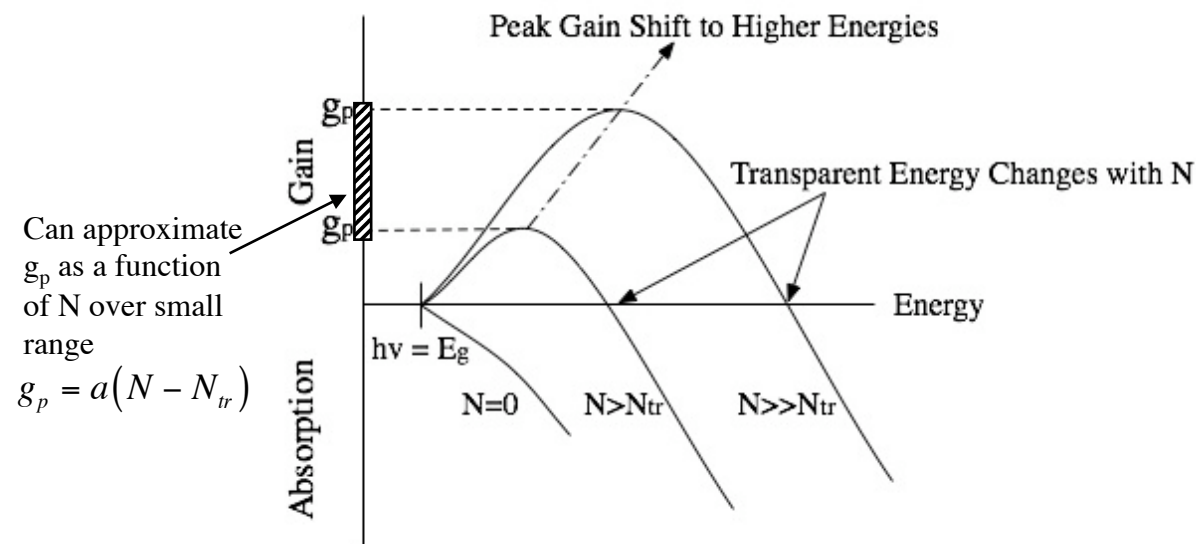


Optical Gain in Semiconductors (3)

- Optical gain by stimulated emission for a photon of energy $h\nu$ exceeds band to band absorption if

$$E_{Fc} - E_{Fv} \geq h\nu = E_2 - E_1 \geq E_g$$

- The gain is wavelength dependent with a peak gain that shifts as a function of carrier density.



Optical Gain in Semiconductors (4)

- The injected carrier density N is determined by the laser current I , the recombination rate $R(N)$ and the active region volume V

$$I = qR(N)V$$

$$R(N) = \frac{N}{\tau_s} + BN^2 + CN^3$$

$$\tau_n = \frac{N}{R(N)}$$

where $A=N/\tau_s$ is the linear non-radiative recombination rate, B is the radiative bimolecular (band-to-band) recombination rate and C is the non-radiative Auger recombination rate

- Note: We will see that while the gain is coupled to the carrier density, the carrier density is coupled to the photon density and therefore to the gain. This coupling will lead to nonlinear gain or gain saturation, as will be discussed later in the carrier rate equations.

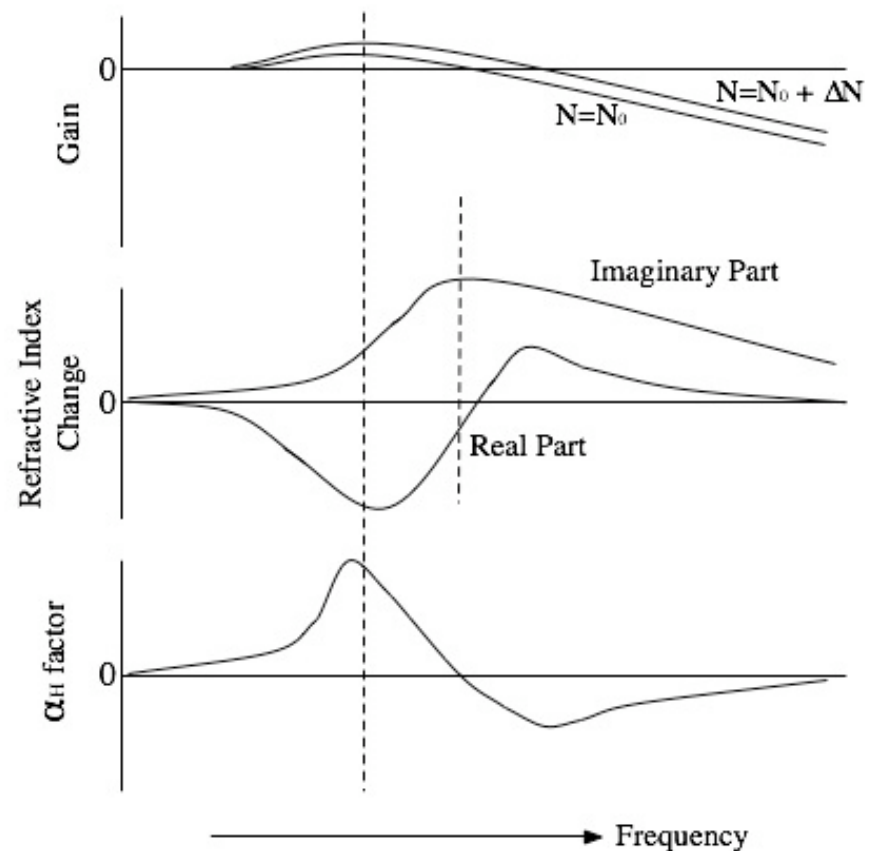
Coupling between Optical Gain and Phase

- The Kramers-Kronig relations tell us that changes in the imaginary (Δg) and real parts of the refractive index (Δn) are related by

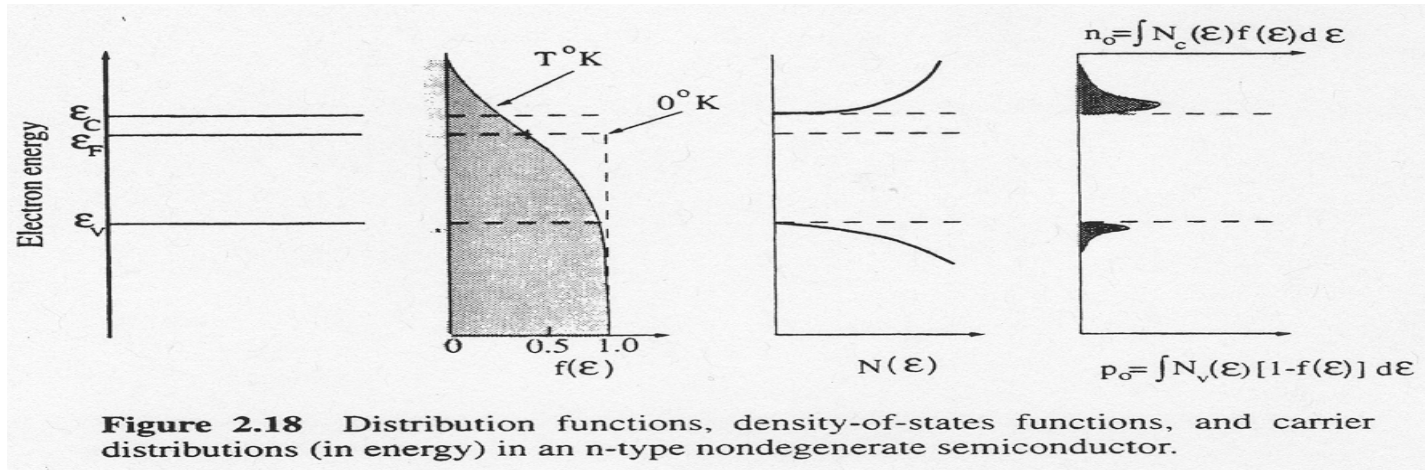
$$\Delta n'(\omega) = \frac{c}{\pi} P \int_0^\infty \frac{\Delta g(\omega')}{\omega'^2 - \omega^2} d\omega'$$

- The coupling between gain and phase is described by the linewidth enhancement (or alpha) factor

$$\alpha_H = -\frac{\partial n' / \partial N}{\partial n'' / \partial N}$$



Gain

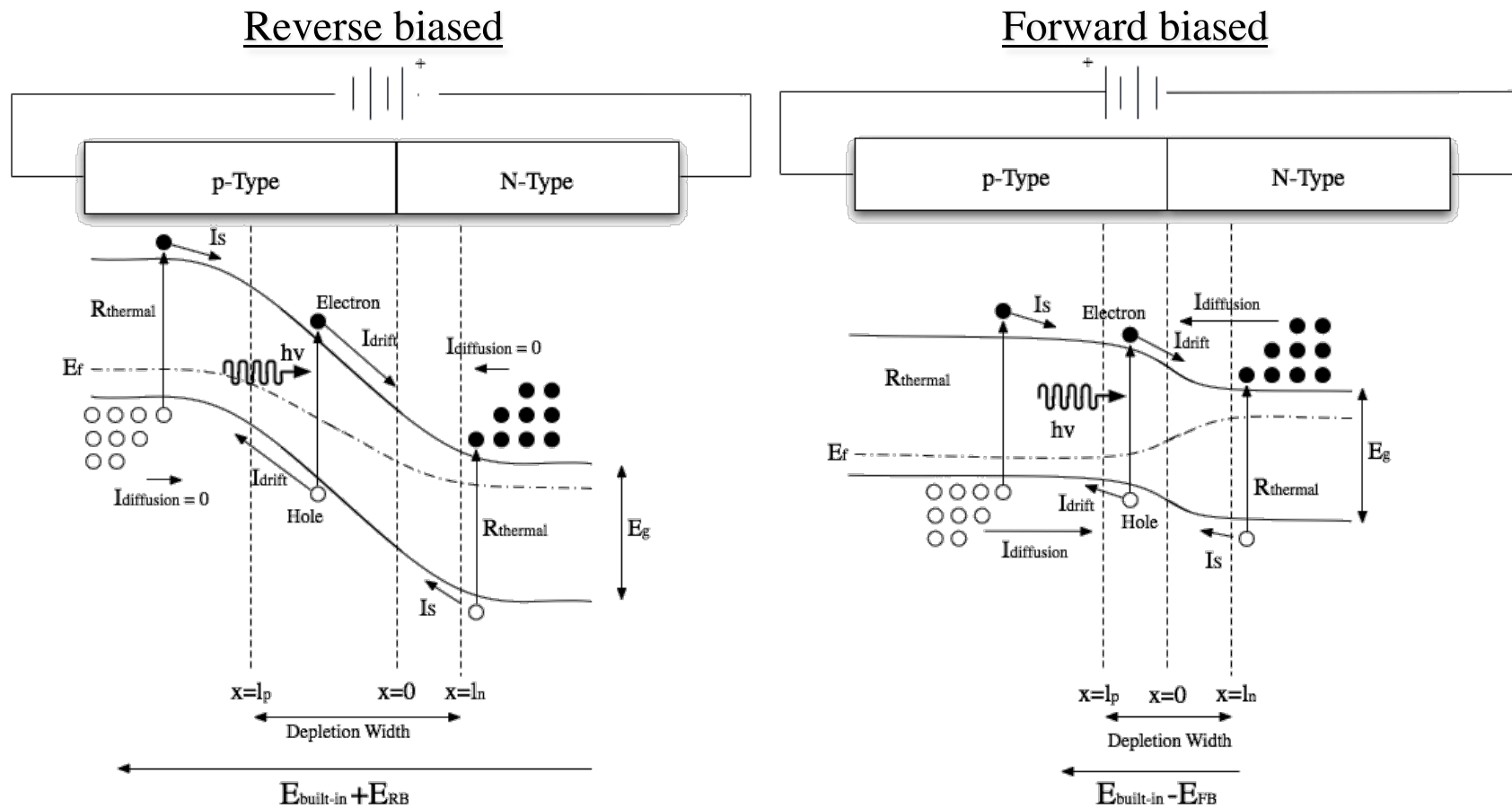


High gain requires

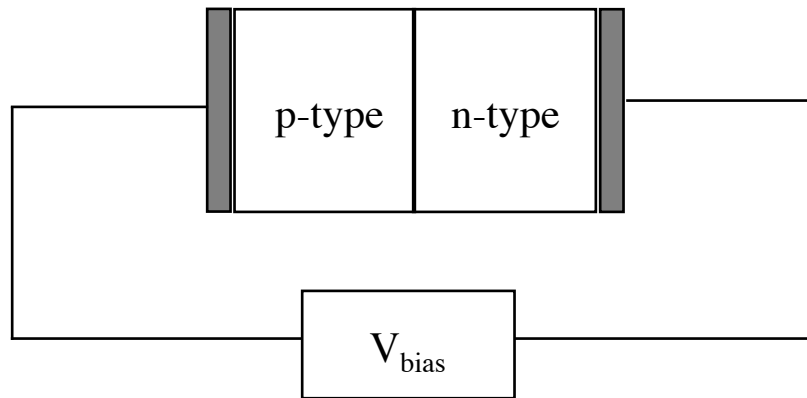
- 1) upper level full ($f \sim 1$)
- 2) lower level empty ($f \sim 0$)

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

Biased p-n Junction Photodiodes



p-n Junction Equation



$$I = (I_s) [\exp^{qV_{\text{bias}}/K_B T} - 1]$$

- $I_s = I_{\text{th}}$ is the thermal or saturation current that occurs in normal (non-illuminated) diode operating mode
- q is the electron charge
- V_{bias} is applied bias voltage (positive = forward, negative=reverse)
- K_B is Boltzman's constant
- T is temperature (usually in Kelvin, depending on units of K_B)

Carrier Injection



- In equilibrium,

$$pn = n_i^2$$

- Under forward bias,

$$pn \gg n_i^2$$

- Under reverse bias,

$$pn \ll n_i^2$$

Carrier Injection



- In equilibrium,

$$pn = n_i^2$$

$$E_{Fn} = E_{Fp} = E_F$$

- Under forward bias,

$$pn \gg n_i^2$$

$$E_{Fn} - E_{Fp} > 0$$

- Under reverse bias,

$$pn \ll n_i^2$$

$$E_{Fn} - E_{Fp} < 0$$

Quasi Fermi Levels



$$n = n_i e^{(E_{Fn} - E_i) / kT}$$

$$p = n_i e^{-(E_{Fp} - E_i) / kT}$$

$$pn = n_i^2 e^{(E_{Fn} - E_{Fp}) / kT}$$

Quasi Fermi Levels

$$n = n_i e^{(E_{Fn} - E_i)/kT}$$

$$p = n_i e^{-(E_{Fp} - E_i)/kT}$$

$$pn = n_i^2 e^{(E_{Fn} - E_{Fp})/kT}$$

- Gain occurs when

$$g(\hbar\omega) > 0 \quad \text{when} \quad E_{Fn} - E_{Fp} > \hbar\omega$$

Optical Gain in Semiconductors

Gain between two levels depends on:

- Carrier density, i.e. level of inversion

$$\frac{R_{21}}{R_{12}} = \frac{f_2(1-f_1)}{f_1(1-f_2)} = e^{(\Delta E_f - E_{21})/kT}$$

- Reduced density of states

$$\frac{1}{\rho_r} = \frac{1}{\rho_c} + \frac{1}{\rho_v}$$

- Transition matrix element $|M|^2$

Gain : Reduced Density of States

The optical gain is proportional to the reduced density of state at the transition energy:

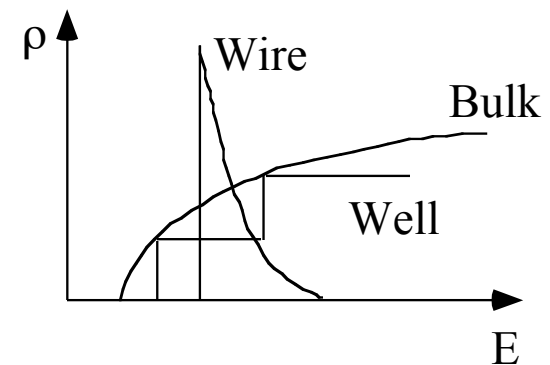
$$g(\omega) \propto (f_2 - f_1) \rho_r(\hbar\omega)$$

The reduced density of states:

$$\rho_r(E) = \frac{\sqrt{E}}{4\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} \quad (\text{Bulk})$$

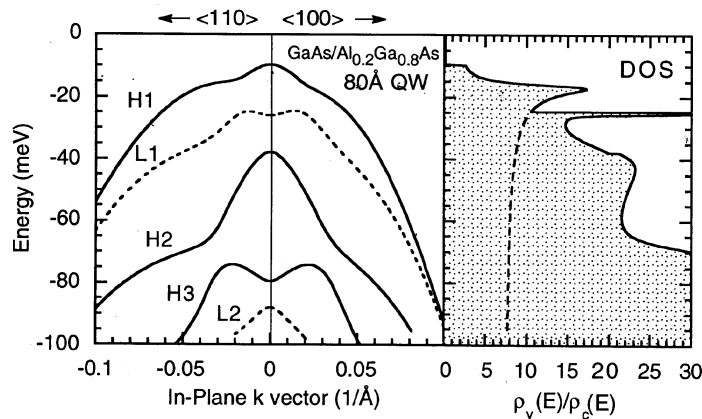
$$\rho_r(E) = \frac{m}{2\pi\hbar^2} \frac{1}{L_z} \quad (\text{Well})$$

$$\rho_r(E) = \frac{\sqrt{2m}}{\hbar} \frac{1}{2\pi L_x L_y \sqrt{E}} \quad (\text{Wire})$$

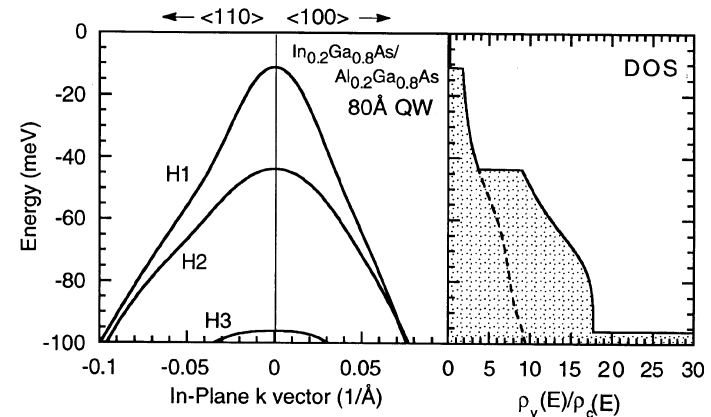


Gain : Strained QW

Compressive strain increases the energy gap between the heavy hole and the light hole subbands. This means fewer carriers in light hole band.



Lattice matched QW

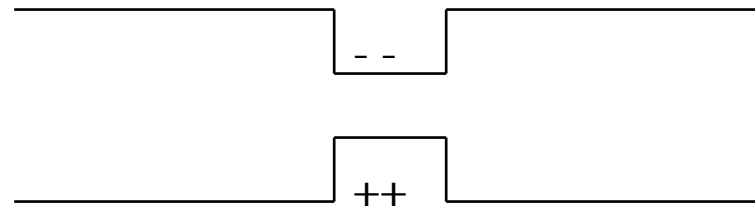


Compressive strain

L.A. Coldren and S.W. Corzine, "Diode Lasers and Photonic Integrated Circuit", Wiley (1995)

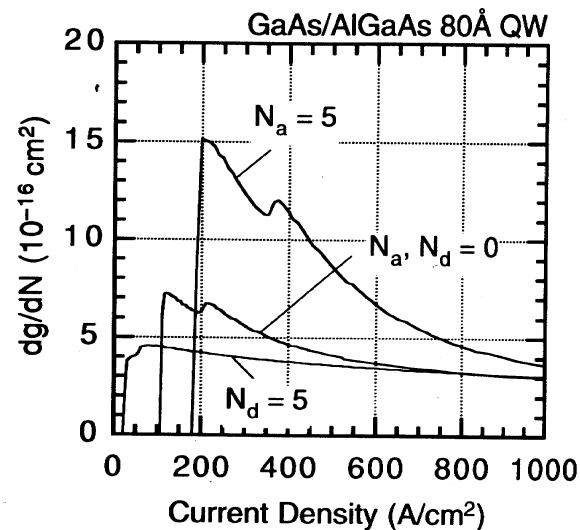
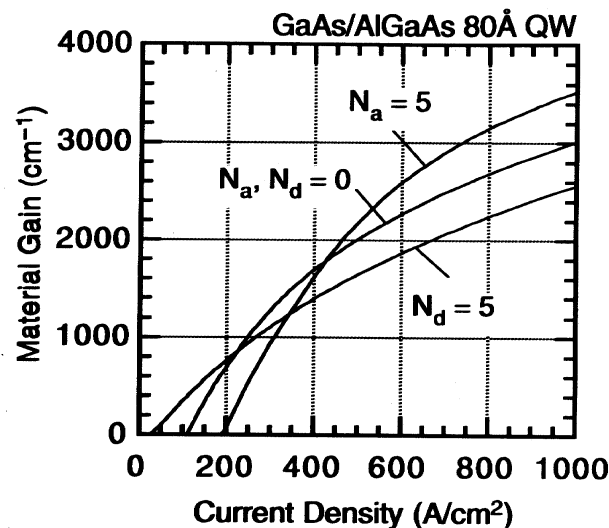
Double Heterostructure Lasers (Kroemer)

- Carriers diffuse away so it is difficult to get high gain
- A method of confining the carriers to a region in space is necessary
- Double heterostructure (proposed in 1964 but not implemented until 1968, which led to the first cw lasers).



Gain : Doping

p-doping in the active region increases the differential gain, at the expense of increased threshold current density



L.A. Coldren and S.W. Corzine, "Diode Lasers and Photonic Integrated Circuit", Wiley (1995)

Rate Equations

Neglecting the phase of the optical field, the length dependence of the carrier and photon densities, and the modal dependence; the rate equations for the averaged photon and carrier densities become:

$$\frac{dS}{dt} = \frac{\Gamma v_g a (N - N_{tr})}{1 + \epsilon S} S - \frac{S}{\tau_p} + \frac{\beta \Gamma N}{\tau_n}$$

$$\frac{dN}{dt} = \eta_i \frac{I}{qV} - \frac{v_g a (N - N_{tr})}{1 + \epsilon S} S - \frac{N}{\tau_n}$$